Problem 1. ***(DPV Exercise 7.1: 20 points)***
See Figure 1. Let $obj(x,y) = 5x+3y$. Then $obj(0,0) = 0$, $obj(5,0) = 25$, $obj(5,2) = 31$, and $obj(2,5) = 25$. So the max is 31 at $(5,2)$.

Problem 2. ***(DPV Exercise 7.4: 20 points)***
Variables: x_r = pints of regular Duff ordered and x_s = pints of strong Duff ordered.
Objective function to maximize: $1 \cdot x_r + 1.5 \cdot x_s$.
Constraints: $2x_s \leq x_r$, $x_r + x_s \leq 3000$, $x_r, x_s \geq 0$.
See Figure 2. Let $obj(x_r,x_s) = x_r + 1.5 \cdot x_s$. Then: $obj(0,0) = 0$, $obj(3000,0) = 3000$, and $obj(2000,1000) = 3500$. So \$3,500 is the maximum at $(2000,1000)$.

Problem 3. ***(DPV Exercise 7.6: 8 points)***
Minimize: $x+y$ under constraints $x,y \geq 0$. The feasible region is the entire 1st quadrant, but the $x+y$ has a unique minimal value at $(0,0)$.

Problem 4. ***(DPV Exercise 7.7: 12 points)***
- **(a)** In order to be infeasible, the half-plane described by $ax+by \leq 1$ does not intersect the 1st quadrant. But if $x=y=0$, then $0 = ax+by \leq 1$. So there always a feasible point.
- **(b)** $a \leq 0$ or $b \leq 0$.
- **(c)** $a > 0$ and $b > 0$.

Problem 5. ***(DPV Exercise 7.17: 20 points)***
- **(a)** 11 is the max-flow.
  $(\{S,A,B\}, \{C,D,T\})$ is the min-cut.
- **(b)** See Figure 3. The nodes reachable from S are in funny circles and the nodes reachable from T are in funny squares.
- **(c)** (A,C) and (B,C).
- **(d)** See Figure 4.
- **(e)** Construct the residual graph and do a graph search that looks for edges with residual capacity 0 but have edges on both ends of them with positive residual capacity.

Problem 6. ***(DPV Exercise 7.18 parts (a) and (b): 20 points)***
- **(a)** From the original graph $G = (V,E)$, construct a graph $G' = (V',E')$ where:

$$V' = V \cup \{S_*, T_*\}.$$
$$E' = E \cup \{(S_*,s)) : s \text{ is a source in } G\} \cup \{(t,T_*)) : t \text{ is a target in } G\}$$

  Each edge from E has the same capacity as in G. Each (S_*,s) and (t,T_*) edge has capacity C where C is, say, the sum of all the capacities in G. S_* and T_* are the source and target nodes in G', respectively.
- **(b)** From the original graph $G = (V,E)$, construct a graph $G' = (V',E')$ where:

$$V' = V \cup \{\hat{v} : v \in V\}.$$
$$E' = \{(u,\hat{v}) : (u,v) \in E\} \cup \{(v,\hat{v}) : v \in V\}.$$

  Each $(u,\hat{v})$ has the same capacity as (u,v) in G. Each $(\hat{v},v)$ has the max-capacity of v. $\hat{S}$ is the source node for G'.

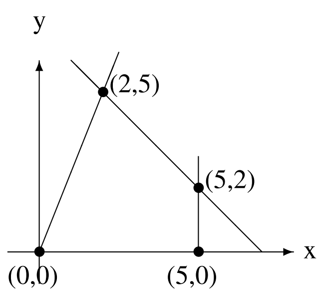

Figure 1

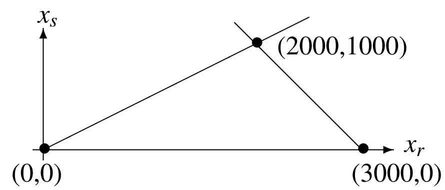

Figure 2

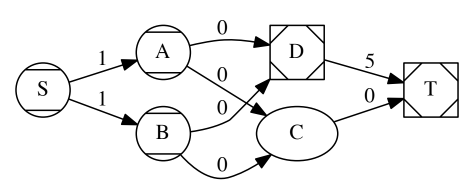

Figure 3

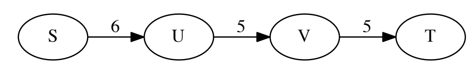

Figure 4