Abbreviations. *IH* = Induction Hypothesis. *Math Fact n* = Math Fact n in the Big-O writeup. *TLR* = The Limit Rule (from the Big-O writeup).

1. PG Problem 27. ***Corrected:.***
BASE CASE. $0^3+2\cdot 0=0$ and 3 divides 0.
INDUCTION STEP. *IH:* 3 divides n^3+2n. *Goal:* Show 3 divides $(n+1)^3+2(n+1)$. By some algebra, $(n+1)^3+2(n+1) = n^3+3n^3+3n+1+2n+2 = (n^3+2n)+3(n^2+n+1)$. Since 3 divides both n^3+2n and $3(n^2+n+1)$, 3 also divides $(n+1)^3+2(n+1)$. So we are done.

2. PG Problem 176.
Since $f_1 \in O(g_1(n))$ and $f_2 \in O(g_2(n))$, there are constants n_1, n_2, c_1, and c_2 such that:

i) for all $n \geq n_1$, $f_1(n) \leq c_1\cdot g_1(n)$ and
ii) for all $n \geq n_2$, $f_2(n) \leq c_2\cdot g_2(n)$.

Let $n_0 = \max(n_1, n_2)$ and $c_0 = c_1\cdot c_2$ Then, for all $n \geq n_0$,

$$\begin{aligned} f_1(n)\cdot f_2(n) &\leq (c_1\cdot g_1(n))\cdot(c_2\cdot g_2(n)) \\ &\leq (c_1\cdot c_2)\cdot(g_1(n)\cdot g_2(n)) \\ &= c_0\cdot(g_1(n)\cdot g_2(n)). \end{aligned}$$

Therefore, $f_1(n)\cdot f_2(n)$ is in $O(g_1(n)\cdot g_2(n))$.

3. PG Problem 178.
Let $f(n)=n$ and $g(n) = \begin{cases} n^2, & \text{if } n \text{ is even;} \\ 1, & \text{if } n \text{ is odd.}\end{cases}$ So:

$$\lim_{n\to\infty}\frac{g(2n)}{f(2n)} = \lim_{n\to\infty} 2n = \infty. \qquad \lim_{n\to\infty}\frac{f(2n+1)}{g(2n+1)} = \lim_{n\to\infty}(2n+1) = \infty.$$

Thus, $g \notin O(f(n))$ and $f \notin O(g(n))$.

4. DPV Exercise 0.1.
Part a. $\lim_{n\to\infty}\frac{n-100}{n-200} = 1$. So by TLR, f is in $\Theta(g)$.
Part b. $\lim_{n\to\infty}\frac{n^{1/2}}{n^{2/3}} = \lim_{n\to\infty} n^{-1/6} = 0$. So by TLR, f is in $O(g)$.
Part c. $\lim_{n\to\infty}\frac{100n+\log n}{n+(\log n)^2} = \lim_{n\to\infty}\frac{100+(\log n)/n}{1+(\log n)^2/n} = 100$. So by TLR, f is in $\Theta(g)$.
Part e. $\lim_{n\to\infty}\frac{\log 2n}{\log 3n} = \lim_{n\to\infty}\frac{(\log n)+(\log 2)}{(\log n)+(\log 3)} = 1$. So by TLR, f is in $\Theta(g)$.
Part f. $\lim_{n\to\infty}\frac{10\log n}{\log(n^2)} = \lim_{n\to\infty}\frac{10\log n}{2\log n} = 5$. So by TLR, f is in $\Theta(g)$.
Part h. $\lim_{n\to\infty}\frac{n^2/\log n}{n(\log n)^2)} = \lim_{n\to\infty}\frac{n}{(\log n)^3)} = \infty$, by Math Fact 6(a). So by TLR, f is in $\Omega(g)$.
Part l. First note that $5^{\log_2 n} = (2^{\log_2 5})^{\log_2 n} = 2^{(\log_2 5)\cdot(\log_2 n)} = 2^{(\log_2 n)\cdot(\log_2 5)} = (2^{\log_2 n})^{\log_2 5} = n^{\log_2 5}$ and $\log_2 5 \approx 2.322$. Hence, $\lim_{n\to\infty}\frac{n^{1/2}}{5^{\log_2 n}} = \lim_{n\to\infty}\frac{n^{1/2}}{n^{\log_2 5}} = \lim_{n\to\infty} n^{1/2-(\log_2 5)} = 0$, since $\frac{1}{2}-\log_2 5 < -1.8$. So by TLR, f is in $O(g)$.
Part m. $\lim_{n\to\infty}\frac{n2^n}{3^n} = \lim_{n\to\infty} n(\frac{2}{3})^n = \lim_{n\to\infty}\frac{n}{(\frac{3}{2})^n} = 0$, by Math Fact 6(b). So by TLR, f is in $O(g)$.

5. DPV Exercise 0.2.
By Math Fact 5, when $c \neq 1$, then $g(n) = 1+c+c^2+\cdots+c^n = \frac{c^{n+1}-1}{c-1}$.
CASE: $0<c<1$. Then $c \geq c^{n+1}$ for all n. Hence, for all n, $\frac{c^{n+1}-1}{c-1} \leq 1$. Thus, $g(n)\in\Theta(1)$.
CASE: $c=1$. Then $g(n)=n+1$. Thus, $g(n)\in\Theta(n)$.
CASE: $c>1$. Then $g(n) = \frac{c^{n+1}-1}{c-1} \in \Theta(c^{n+1})$.

6. DPV Exercise 0.3a
BASE CASES: $F_6 = 8 = 2^{6/2}$ and $F_7 = 13 \geq 11.314 > 2^{7/n}$.
INDUCTION STEP. *IH:* $F_n \geq 2^{n/2}$ and $F_{n+1} \geq 2^{(n+1)/2}$ for $n \geq 6$. We need to show that $F_{n+2} \geq 2^{(n+2)/2}$. We note that:

$$F_{n+2} = F_{n+1}+F_n \geq 2^{(n+1)/2}+2^{n/2} \geq 2^{n/2}+2^{n/2} = 2\cdot 2^{n/2} = 2^{(n/2)+1} = 2^{(n+2)/2}.$$

7. PG Problem 52.
BASE CASE. For $n=1$, $\sum_{i=0}^{1}F_i = F_0+F_1 = 0+1 = 1 = 2-1 = F_3-1$.
INDUCTION STEP: *IH:* $\sum_{i=0}^{n}F_i = F_{n+2}-1$ for $n>0$.
We need to show $\sum_{i=0}^{n+1}F_i = F_{n+3}-1$. We first note that $F_{n+1}+F_{n+2} = F_{n+3}$. Now:

$$\begin{aligned} \textstyle\sum_{i=0}^{n+1}F_i &= (\textstyle\sum_{i=0}^{n}F_i)+F_{n+1} \\ &= (F_{n+2}-1)+F_{n+1} && \text{(by the IH)} \\ &= (F_{n+1}+F_{n+2})-1 \\ &= F_{n+3}-1. \end{aligned}$$